SUSTAINABILITY ASSESSMENT PROGRAM

UNSW AUSTRALIA | Water Research Centre
School of Civil and Environmental Engineering

SAP Working Paper
October 2016

Note on the Decomposition of Total Impact Multipliers in a Supply-and-Use Framework

Thomas Wiedmann

Sustainability Assessment Program (SAP), Water Research Centre
School of Civil and Environmental Engineering
UNSW Australia, Sydney, NSW 2052, Australia

1 Introduction

Total Impact Multipliers (TIMs) are factors derived from input-output analysis (IOA) that show the total, economy-wide attribution of impacts from production to one unit of final demand. One example is the attribution of greenhouse gas (GHG) emissions of all industries in an economy to the final demand of a particular product. This metric is called the carbon footprint of the product (Gao et al., 2014; Peters, 2010; Wiedmann, 2009). More precisely, is the cradle-to-sale, life-cycle inventory of total upstream GHG emissions released during the production of the product.

TIMs are the typical result of a demand-pull, Leontief Type I calculation in the standard Leontief quantity model (Miller and Blair, 2009, p.447):

$$\mathbf{TIMs} = \mathbf{DIMs} \cdot (\mathbf{I} - \mathbf{A})^{-1} = \mathbf{DIMs} \cdot \mathbf{L} \qquad \text{Eq. 1}$$

where

DIMs is a matrix of Direct Impact Multipliers, i.e. factors that represent the direct impact intensity q of an industry (or sector) in the form of the total impact Q of this industry divided by the industry's total output X (i.e. $\mathbf{q} = \mathbf{Q} \cdot \hat{\mathbf{X}}^{-1}$).

I is an identity matrix with ones on the diagonal and zeroes elsewhere.

A is the technology coefficient matrix, calculated as the product of the input-output transaction matrix **T** and the inverse matrix $\mathbf{X}^{-1}$ of diagonalised total industry output.

L is the standard Leontief Inverse.

TIMs is the resulting matrix of Total Impact Multipliers with the same dimensions as the **DIMs** matrix.

To provide detailed information regarding the origins of impacts along the production (supply) chain of a product, its TIM can be decomposed into contributions from sectors involved directly or indirectly in the production. It may be of interest to know in detail which industries or which products contribute the most to the total impact, for example, if the aim is to reduce impacts (e.g. climate mitigation, resource efficiency, cost savings etc) or to increase factors of production (e.g. employment, profits, etc.). In the following, this paper will distinguish between impact contributions from industries and products and elaborate on two different ways of decomposing TIMs in a Supply-and-Use-Table (SUT) framework.

An SUT framework shows the sales of products to industries (intermediate demand) in the Use Table part and the value of products produced by industries in the Supply Table part (Eurostat, 2008). It can be regarded superior to a symmetric (industry-by-industry or product-by-product) input-output table, in the sense that more original information on sales and production structures is preserved (Rueda-Cantuche, 2011). In particular, information on co- or by-production is valuable for applications in industrial ecology and related fields. Lenzen and Rueda-Cantuche (2012) demonstrate how, in a SUT framework, impact satellite data can be assigned to both industries and products and that total impact multipliers are obtained for both entities.

Whilst impact analyses have been performed in SUT frameworks many times (e.g. Fry et al., 2015; Kagawa and Suh, 2009; Lenzen et al., 2004; Malik et al., 2016; Malik et al., 2014; Suh et al., 2010; Wachsmann et al., 2009; Wiedmann et al., 2006; Wiedmann et al., 2011) and whilst the decomposition of TIMs has been described in general terms before (Nakamura and Nansai, 2016, section 3.7.1), a decomposition that explicitly distinguishes industries and products in a generalised SUT framework has – to the knowledge of the author – not been described yet.

This note provides the mathematical description of SUT-based TIM decompositions in the following section as well as a worked example in the form of an Excel worksheet and Matlab code as Supplementary Material (SM). Results from the worked example are presented in the figures.

2 Methods for decomposing TIMs

This section describes two types of decomposition – by industry and by products – and explains the differences. The framework chosen is a generalised SUT system with m industries and n products, which may have square (m=n) or rectangular (m≠n) supply and use tables.

Figure 1 shows the example SUT data with environmental extensions. Note that in this framework, the supply table is transposed with industries in rows and products in columns. This is usually referred to as a "make" matrix (Eurostat, 2008). Because of its widespread use, however, the acronym SUT is retained throughout this text.

	Ind A	Ind B	Ind C	Ind D	Ind E	Ind F	Prod 1	Prod 2	Prod 3	Prod 4	Prod 5	Prod 6	Prod 7	Prod 8	Prod 9	Prod 10	Final demand (y)	**Total output (X')**	Units
Ind A	-	-	-	-	-	-	320	205	30	-	-	-	-	-	-	-	-	**555**	$m
Ind B	-	-	-	-	-	-	-	-	310	400	200	-	-	-	-	-	-	**910**	$m
Ind C	-	-	-	-	-	-	-	-	-	10	330	490	100	-	-	-	-	**930**	$m
Ind D	-	-	-	-	-	-	-	-	-	-	-	-	400	700	-	-	-	**1,100**	$m
Ind E	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1,800	-	-	**1,800**	$m
Ind F	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1,700	-	**1,700**	$m
Prod 1	20	30	40	35	55	20	-	-	-	-	-	-	-	-	-	-	120	**320**	$m
Prod 2	-	25	10	35	25	30	-	-	-	-	-	-	-	-	-	-	80	**205**	$m
Prod 3	100	10	10	10	50	50	-	-	-	-	-	-	-	-	-	-	110	**340**	$m
Prod 4	10	200	20	-	20	20	-	-	-	-	-	-	-	-	-	-	140	**410**	$m
Prod 5	50	20	200	-	40	40	-	-	-	-	-	-	-	-	-	-	180	**530**	$m
Prod 6	-	20	-	310	50	-	-	-	-	-	-	-	-	-	-	-	110	**490**	$m
Prod 7	10	5	30	10	200	100	-	-	-	-	-	-	-	-	-	-	145	**500**	$m
Prod 8	-	25	20	20	200	200	-	-	-	-	-	-	-	-	-	-	235	**700**	$m
Prod 9	10	25	10	5	300	500	-	-	-	-	-	-	-	-	-	-	950	**1,800**	$m
Prod 10	80	5	5	-	50	400	-	-	-	-	-	-	-	-	-	-	1,160	**1,700**	$m
Value added	275	545	585	675	810	340	-	-	-	-	-	-	-	-	-	-			$m
Total input (X)	**555**	**910**	**930**	**1,100**	**1,800**	**1,700**	**320**	**205**	**340**	**410**	**530**	**490**	**500**	**700**	**1,800**	**1,700**			$m
																		Total E	
Direct emissions (E)	100	300	300	100	1,000	1,700	-	-	-	-	-	-	-	-	-	-		**3,500**	kt
DIMs (f = E / X)	0.180	0.330	0.323	0.091	0.556	1.000	-	-	-	-	-	-	-	-	-	-			kg/$

Figure 1: Example asymmetric make (supply) and use table for six industries and ten products with extensions for industrial greenhouse gas emissions (all numbers are fictitious; zeros are depicted with a hyphen)

2.1 Decomposing TIMs by industry

A decomposition of a product's impact multiplier by *industry* answers the question: How large is the impact from one particular industry involved in all of the production steps of this particular product? The decomposition shows the contribution of an industry as the ultimate emissions source in the whole production/supply chain of the product. An example are the emissions from electricity as part of the total carbon footprint of a product. This refers to the total use of electricity during the production of the product, independent of which process or industry actually uses the electricity (most likely, electricity was used in virtually all steps of the product's cradle-to-shelf life cycle).

A decomposition by industry is achieved by creating a diagonal matrix of direct impact multipliers (DIMs) which is post-multiplied with the Leontief Inverse:

$$\mathbf{TIMs}^{\otimes} = \widehat{\mathbf{DIMs}} \cdot \mathbf{L} \qquad \text{Eq. 2}$$

where:

$\mathbf{TIMs}^{\otimes}$ is a (m+n)×(m+n) matrix of TIMs decomposed by industry; column sums add up to total TIMs for industries and products

$\widehat{\mathbf{DIMs}}$ is a (m+n)×(m+n) matrix with DIMs placed on the diagonal of a (m+n)×(m+n) matrix of zeros. The hat symbol ($\widehat{\ }$) denotes the diagonalization which is accomplished by row-wise multiplication (symbol ×) of the row vector of impact intensities **q** with the identity matrix:

$$\widehat{\mathbf{DIMs}} = \mathbf{q} \times \mathbf{I} \qquad \text{Eq. 3}$$

L is the Leontief Inverse of the SUT with dimensions (m+n)×(m+n)

This method recognises the ultimate origin of impacts making up the life-cycle inventory of a product. All the possible supply chain paths that start with industry *i* (and its impact) and end with product *p* sum up to the total share of industry *i* in the TIM of product *p* (independent of how long these paths may be). This information is useful if a particular industry is targeted in an impact reduction strategy. For example, if direct emissions from electricity generation become zero because a whole country's electricity comes from 100% renewable power, then the carbon footprint intensity (TIMs) of all products are reduced by the electricity sector's contribution of the industry-decomposed TIMs.

Decomposition of TIMs by INDUSTRY: column sums add up to total TIMs:

																	Units
Total TIMs	**0.649**	**0.568**	**0.530**	**0.299**	**0.906**	**1.801**	**0.649**	**0.649**	**0.576**	**0.568**	**0.545**	**0.530**	**0.346**	**0.299**	**0.906**	**1.801**	kg/$
	Ind A	Ind B	Ind C	Ind D	Ind E	Ind F	Prod 1	Prod 2	Prod 3	Prod 4	Prod 5	Prod 6	Prod 7	Prod 8	Prod 9	Prod 10	
Ind A	0.199	0.018	0.016	0.018	0.019	0.021	0.199	0.199	0.034	0.018	0.017	0.016	0.018	0.018	0.019	0.021	kg/$
Ind B	0.116	0.446	0.068	0.032	0.045	0.059	0.116	0.116	0.417	0.437	0.211	0.068	0.039	0.032	0.045	0.059	kg/$
Ind C	0.046	0.033	0.390	0.117	0.065	0.068	0.046	0.046	0.034	0.041	0.255	0.390	0.172	0.117	0.065	0.068	kg/$
Ind D	0.009	0.006	0.007	0.096	0.025	0.031	0.009	0.009	0.006	0.006	0.007	0.007	0.078	0.096	0.025	0.031	kg/$
Ind E	0.063	0.033	0.019	0.013	0.686	0.272	0.063	0.063	0.036	0.033	0.025	0.019	0.015	0.013	0.686	0.272	kg/$
Ind F	0.216	0.032	0.028	0.023	0.067	1.350	0.216	0.216	0.048	0.032	0.030	0.028	0.024	0.023	0.067	1.350	kg/$
Prod 1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	kg/$
Prod 2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	kg/$
Prod 3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	kg/$
Prod 4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	kg/$
Prod 5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	kg/$
Prod 6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	kg/$
Prod 7	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	kg/$
Prod 8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	kg/$
Prod 9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	kg/$
Prod 10	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	kg/$

Figure 2: Example decomposition of total impact multipliers by industry

The results of the TIM decomposition are in SUT format, i.e. the contributions of both industries to both, industry and product TIMs, are shown. More precisely (see example in Figure 2), the left part of the TIM decomposition matrix, i.e. the first m columns, show the contributions of industries to industry TIMs. In the right part, i.e. the last n columns the industry contributions to product TIMs are shown. It is this latter breakdown of product TIMs that would usually be used for further footprint calculations, since it is products for which there is a (final) demand, not industries. Note that if a product is only produced by one industry, the TIM of this product and the corresponding industry TIM are identical (for example Ind A and Prod 1 and 2 in Figure 2).

2.2 Decomposing TIMs by product

Whilst the previous type of decomposition reveals industry contributions, it does not provide any information on the contribution of products that are required to produce a particular good or service. The latter type of information is often reported in the life-cycle inventory (LCI) of products or processes and of high relevance if, for example, one product in the production chain is replaced by a different product with a different impact (e.g. normal concrete with low-carbon concrete).

A decomposition of a product's impact multiplier by *product* answers the question: What are the life-cycle impacts of those products that are directly used in the production of a good or service? In other words, the indirect impacts that are *embodied* in products that act as inputs to the industry that produces said good or service.

One example may be the material composition of wind turbines, which are mostly made of steel, plastic, concrete and copper (amongst other, minor materials). A decomposition by product identifies the full, life-cycle contribution of these materials. If steel is fully replaced by a different material, then the contribution of steel to the wind turbine's TIM becomes zero.

Mathematically the decomposition by product proceeds via the isolation of inputs of products *m* to the production of product *n*. In the technical coefficient matrix A these are all elements $a_{m,n}$ in the column of industry n, constituting the final stage of all inputs needed by industry n (to produce product n). Using the power series approximation (also called Taylor expansion or Neumann series; Waugh, 1950; Miller and Blair, 2009, section 2.4), we can write:

$$\mathbf{TIMs} = \mathbf{qI} + \mathbf{qA} + \mathbf{qA}^2 + \mathbf{qA}^3 + \mathbf{qA}^4 + \ldots \qquad \text{Eq. 4}$$

Since we want to know how product m contributes to the TIM of product n we segregate $\mathbf{A}_{m,n}$ thus (compare to Nakamura and Nansai, 2016, section 3.7.1):

$$\mathbf{TIMs} = \mathbf{qI} + \mathbf{qA}_{m,n} + \mathbf{qAA}_{m,n} + \mathbf{qA}^2\mathbf{A}_{m,n} + \mathbf{qA}^3\mathbf{A}_{m,n} + \ \ldots \qquad \text{Eq. 5}$$

$$\mathbf{TIMs} = \mathbf{qI} + \mathbf{q}(\mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \ \ldots) \cdot \mathbf{A}_{m,n} \qquad \text{Eq. 6}$$

$$\mathbf{TIMs} = \mathbf{qI} + \mathbf{q}(\mathbf{I} - \mathbf{A})^{-1} \cdot \mathbf{A}_{m,n} \qquad \text{Eq. 7}$$

$$\mathbf{TIMs} = \mathbf{qI} + \mathbf{q}(\mathbf{I} - \mathbf{A})^{-1} \cdot \mathbf{A}_{m,n} \qquad \text{Eq. 8}$$

from which we can derive $\mathbf{TIMs}^{\oplus}$ as a (m+n)×(m+n) matrix of product contributions:

$$\mathbf{TIMs}^{\oplus} = \widehat{\mathbf{DIMs}} + \widehat{\mathbf{TIMs}} \cdot \mathbf{A}_{\mathbf{m,n}} \qquad \text{Eq. 9}$$

Again, the initial results of the TIM decomposition are in SUT format, i.e. it shows the decomposition of industry TIMs on the left-hand side (first m columns) as well as the decomposition of product TIMs n the right-hand side (last n columns). And again, we are interested in the breakdown of the product TIMs, since footprint calculations are based on multiplying expenditure data with product TIMs, not industry TIMs.

So far, however, the breakdown of product TIMs only shows the contributions from industries. An additional step is necessary to achieve a product-by-product decomposition. If a product is only produced by one industry, then (as stated above) the product TIM and the industry TIM are identical and therefore the product decomposition of the corresponding industry TIM is the result we are looking for: it also constitutes the product decomposition of the product TIM. For example, the column for Ind A adds up to the TIM for Prod 1 (and Prod 2), see Figure 3.

From script TIM_decomposition_mxn_SUT.m
Decomposition of TIMs by PRODUCT
Initial decomposition, distinguishing industry and product contributions

	Ind A	Ind B	Ind C	Ind D	Ind E	Ind F	Prod 1	Prod 2	Prod 3	Prod 4	Prod 5	Prod 6	Prod 7	Prod 8	Prod 9	Prod 10	Units
Total TIMs	**0.649**	**0.568**	**0.530**	**0.299**	**0.906**	**1.801**	**0.649**	**0.649**	**0.576**	**0.568**	**0.545**	**0.530**	**0.346**	**0.299**	**0.906**	**1.801**	kg/$
Ind A	0.180	-	-	-	-	-	0.649	0.649	0.057	-	-	-	-	-	-	-	kg/$
Ind B	-	0.330	-	-	-	-	-	-	0.518	0.555	0.215	-	-	-	-	-	kg/$
Ind C	-	-	0.323	-	-	-	-	-	-	0.013	0.330	0.530	0.106	-	-	-	kg/$
Ind D	-	-	-	0.091	-	-	-	-	-	-	-	-	0.240	0.299	-	-	kg/$
Ind E	-	-	-	-	0.556	-	-	-	-	-	-	-	-	-	0.906	-	kg/$
Ind F	-	-	-	-	-	1.000	-	-	-	-	-	-	-	-	-	1.801	kg/$
Prod 1	0.023	0.021	0.028	0.021	0.020	0.008	-	-	-	-	-	-	-	-	-	-	kg/$
Prod 2	-	0.018	0.007	0.021	0.009	0.011	-	-	-	-	-	-	-	-	-	-	kg/$
Prod 3	0.104	0.006	0.006	0.005	0.016	0.017	-	-	-	-	-	-	-	-	-	-	kg/$
Prod 4	0.010	0.125	0.012	-	0.006	0.007	-	-	-	-	-	-	-	-	-	-	kg/$
Prod 5	0.049	0.012	0.117	-	0.012	0.013	-	-	-	-	-	-	-	-	-	-	kg/$
Prod 6	-	0.012	-	0.149	0.015	-	-	-	-	-	-	-	-	-	-	-	kg/$
Prod 7	0.006	0.002	0.011	0.003	0.038	0.020	-	-	-	-	-	-	-	-	-	-	kg/$
Prod 8	-	0.008	0.006	0.005	0.033	0.035	-	-	-	-	-	-	-	-	-	-	kg/$
Prod 9	0.016	0.025	0.010	0.004	0.151	0.267	-	-	-	-	-	-	-	-	-	-	kg/$
Prod 10	0.260	0.010	0.010	-	0.050	0.424	-	-	-	-	-	-	-	-	-	-	kg/$

Note: column sums add up to total TIMs

Figure 3: Interim stage of a TIM decomposition by product. Product information is shown under industry TIMs but has not been assigned to product TIMs yet.

In those cases where one product is produced by two or more industries, the TIMs of these industries (and their product decomposition from) need to be scaled according to the proportion of the industry's contribution to the production of the product. This proportion can simply be derived from the supply (make) table part of the A matrix.

Let $A_{1:m,p}$ be the first m rows of product column p in the A matrix derived from the SUT (which has the dimensions (m+n)×(m+n)). Transposing this column (into one row with m values) and row-wise multiplying it with the industry TIM columns from the previous step, results in new columns of decomposed industry TIMs that are scaled according to the contribution of all industries to the production of product p. These columns simply need to be added together to result in one column that shows a decomposition of the TIM of product p. This column then contains contributions from both industries (first m rows) and products (last n rows) which can be interpreted as follows (see Figure 4). Contributions from industries are the *direct* emissions from industries that produce product p as a direct output. These include emissions from the 'own' industry where product p is the main product and from other industries where product p is a by-product, as shown in the supply table. Contributions from products are as described above: life-cycle (cradle-to-sale) impact inventories of products needed to produce product p.

Final decomposition of TIMs by PRODUCT (as calculated by script 'TIM_decomposition_mxn_SUT.m')

	Prod 1	Prod 2	Prod 3	Prod 4	Prod 5	Prod 6	Prod 7	Prod 8	Prod 9	Prod 10	
Total TIMs	**0.649**	**0.649**	**0.576**	**0.568**	**0.545**	**0.530**	**0.346**	**0.299**	**0.906**	**1.801**	kg/$
Ind A	0.180	0.180	0.016	-	-	-	-	-	-	-	kg/$
Ind B	-	-	0.301	0.322	0.124	-	-	-	-	-	kg/$
Ind C	-	-	-	0.008	0.201	0.323	0.065	-	-	-	kg/$
Ind D	-	-	-	-	-	-	0.073	0.091	-	-	kg/$
Ind E	-	-	-	-	-	-	-	-	0.556	-	kg/$
Ind F	-	-	-	-	-	-	-	-	-	1.000	kg/$
Prod 1	0.023	0.023	0.022	0.022	0.025	0.028	0.022	0.021	0.020	0.008	kg/$
Prod 2	-	-	0.016	0.018	0.011	0.007	0.018	0.021	0.009	0.011	kg/$
Prod 3	0.104	0.104	0.015	0.006	0.006	0.006	0.005	0.005	0.016	0.017	kg/$
Prod 4	0.010	0.010	0.115	0.122	0.055	0.012	0.002	-	0.006	0.007	kg/$
Prod 5	0.049	0.049	0.015	0.015	0.077	0.117	0.023	-	0.012	0.013	kg/$
Prod 6	-	-	0.011	0.011	0.004	-	0.119	0.149	0.015	-	kg/$
Prod 7	0.006	0.006	0.002	0.002	0.008	0.011	0.005	0.003	0.038	0.020	kg/$
Prod 8	-	-	0.008	0.008	0.007	0.006	0.006	0.005	0.033	0.035	kg/$
Prod 9	0.016	0.016	0.024	0.025	0.015	0.010	0.005	0.004	0.151	0.267	kg/$
Prod 10	0.260	0.260	0.032	0.010	0.010	0.010	0.002	-	0.050	0.424	kg/$

Interpretation: The total impact of Product 1 is: **0.649** kg per $
0.180 kg/$ come from direct production emissions (Ind 1)
0.023 kg/$ come from life-cycle emissions embodied in Prod 1

Figure 4: *Final stage of product TIM decomposition by product. Contributions from industries represent 'own' impacts from the industry (industries) that produce the product.*

3 Concluding remarks

Life cycle assessment and input-output analysis are the two leading methods for attributing environmental, social and other impacts to products, sectors or consumption (Hellweg and Milà i Canals, 2014; Nakamura and Nansai, 2016). To add information and meaning to life-cycle inventory or footprint results, the analysis of contributions from either industries or products has proven an indispensable tool. This can be achieved by using a supply and use framework in input-output or hybrid input-output analysis: Contributions can be shown by either industry origin (where is the source of impacts?) or by product embodiments (what burden carry the products used in the life-cycle or supply chain?). This note describes a simply technique of decomposing total impact multipliers from a SUT modelling framework and provides a practical example.

4 References

Eurostat (2008) *Eurostat Manual of Supply, Use and Input-Output Tables, 2008 edition*. Methodologies and Working Papers. Office for Official Publications of the European Communities, Luxembourg. http://ec.europa.eu/eurostat

Fry, J., Lenzen, M., Giurco, D. and Pauliuk, S. (2015) An Australian Multi-Regional Waste Supply-Use Framework. *Journal of Industrial Ecology*, n/a-n/a. http://dx.doi.org/10.1111/jiec.12376

Gao, T., Liu, Q. and Wang, J. (2014) A comparative study of carbon footprint and assessment standards. *International Journal of Low-Carbon Technologies*, **9**(3), 237-243. http://ijlct.oxfordjournals.org/content/9/3/237.abstract

Hellweg, S. and Milà i Canals, L. (2014) Emerging approaches, challenges and opportunities in life cycle assessment. *Science*, **344**(6188), 1109-1113. http://dx.doi.org/10.1126/science.1248361

Kagawa, S. and Suh, S. (2009) Multistage Process-Based Make-Use System. In: S. Suh, *Handbook of Input-Output Economics in Industrial Ecology*, Series: Eco-Efficiency in Industry and Science, Vol. 23, Chapter 35: 777-800, Springer. http://www.springer.com/earth+sciences/geostatistics/book/978-1-4020-4083-2?detailsPage=toc

Lenzen, M., Pade, L.-L. and Munksgaard, J. (2004) CO_2 Multipliers in Multi-region Input-Output Models. *Economic Systems Research*, **16**(4), 391-412. http://dx.doi.org/10.1080/0953531042000304272

Lenzen, M. and Rueda-Cantuche, J. M. (2012) A note on the use of supply-use tables in impact analyses. *Statistics and Operations Research Transactions*, **36**(2), 139-152. http://www.idescat.cat/sort/sort362/36.2.2.lenzen-cantuche.pdf, http://www.raco.cat/index.php/SORT/article/view/260677

Malik, A., Lenzen, M., Ely, R. N. and Dietzenbacher, E. (2014) Simulating the impact of new industries on the economy: The case of biorefining in Australia. *Ecological Economics*, **107**(0), 84-93. http://dx.doi.org/10.1016/j.ecolecon.2014.07.022

Malik, A., Lenzen, M. and Geschke, A. (2016) Triple bottom line study of a lignocellulosic biofuel industry. *GCB Bioenergy*, **8**(1), 96-110. http://dx.doi.org/10.1111/gcbb.12240

Miller, R. E. and Blair, P. D. (2009) *Input-Output Analysis: Foundations and Extensions; 2nd Edition*. 2nd edition. 31 May 2009. Cambridge University Press. http://www.cambridge.org/millerandblair

Nakamura, S. and Nansai, K. (2016) Input–Output and Hybrid LCA. In: M. Finkbeiner, *Special Types of Life Cycle Assessment*, 219-291, Springer Netherlands, Dordrecht. http://dx.doi.org/10.1007/978-94-017-7610-3_6

Peters, G. P. (2010) Carbon footprints and embodied carbon at multiple scales. *Current Opinion in Environmental Sustainability*, **2**(4), 245-250. http://dx.doi.org/10.1016/j.cosust.2010.05.004

Rueda-Cantuche, J. M. (2011) The choice of type of input-output table revisited: moving towards the use of supply-use tables in impact analysis. *Statistics and Operations Research Transactions*, **35**(1), 21-38. http://www.raco.cat/index.php/SORT/article/view/242561

Suh, S., Weidema, B., Schmidt, J. H. and Heijungs, R. (2010) Generalized Make and Use Framework for Allocation in Life Cycle Assessment. *Journal of Industrial Ecology*, **14**(2), 335-353. http://dx.doi.org/10.1111/j.1530-9290.2010.00235.x

Wachsmann, U., Wood, R., Lenzen, M. and Schaeffer, R. (2009) Structural decomposition of energy use in Brazil from 1970 to 1996. *Applied Energy*, **86**(4), 578-587. http://www.sciencedirect.com/science/article/pii/S0306261908002006

Waugh, F. V. (1950) Inversion of the Leontief Matrix by Power Series. *Econometrica*, **18**(2), 142-154. http://www.jstor.org/stable/1907265

Wiedmann, T. (2009) Carbon Footprint and Input-Output Analysis - An Introduction. *Economic Systems Research*, **21**(3), 175-186. http://dx.doi.org/10.1080/09535310903541256

Wiedmann, T., Minx, J., Barrett, J. and Wackernagel, M. (2006) Allocating ecological footprints to final consumption categories with input-output analysis. *Ecological Economics*, **56**(1), 28-48. http://dx.doi.org/10.1016/j.ecolecon.2005.05.012

Wiedmann, T. O., Suh, S., Feng, K., Lenzen, M., Acquaye, A., Scott, K. and Barrett, J. R. (2011) Application of Hybrid Life Cycle Approaches to Emerging Energy Technologies – The Case of Wind Power in the UK. *Environmental Science & Technology*, **45**(13), 5900-5907. http://dx.doi.org/10.1021/es2007287